



# ADAPTIVE KNOWLEDGE INJECTION FOR MONTE CARLO TREE SEARCH FOR IMPERFECT INFORMATION GAMES

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# Introduction



- Why Study Games?
- Magic The Gathering:
  - Large Amount of Uncertainty
  - Large Branching Factor
  - Tactical Play
  - Disambiguation

# Magic: The Gathering



- 20 Starting Life
- 60 Card Decks From Set of 10,000
- 5 Different Colours: Red, Blue, Green, White, Black
- Different Types Of Cards:
  - Land
  - Creature
  - Instant

# Land



- Produce Mana
- No Cost
- Can Only Play 1 A Turn



Land Card



Tapped Land Card



# Creature



- Used for Combat
- Has Power and Toughness
- Summoning Sickness

Tapped  
Creature





# Attacking & Blocking

6



- 1 Combat Phase Per Turn
- Turn Player Chooses attackers
- Opponent which creatures to block
- Unblocked creatures deal damage to opponents



# Attacking & Blocking

7



- Creatures deal damage equal to their power to enemy creatures toughness simultaneously.

**Deals 1 damage.  
Receives 2 damage**

**Deals 2 damage.  
Receives 1 damage**



# Attacking & Blocking

8





# Attacking & Blocking



- Another Example...



# Attacking & Blocking

10



- Another Example...

**Deals 2 damage.  
Receives 2  
damage**



**Deals 2  
damage.  
Receives 1  
damage**

**Deals 2 damage to  
creature 1 and 1  
damage to  
creature 2.**

**Receives 4  
damage**



# Attacking & Blocking



- Another Example...



# Instant



- Once Off
- Play During any Phase of the game
- Make the game highly interactive



# Magic: The Gathering

13



- Turn Based
- Different Phases
  - Upkeep
  - Draw
  - Main
  - Combat (Attack, Block)
  - Second Main
  - End



# Sample Game

14



- Jeremy has rolled 14
- Bob has rolled 11
- Jeremy Chooses to play first

# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Cards In Hand





# Cards In Hand

17



## Option 1:

- Turn 1: Foundry Street Denizen
- Turn 2: 2 x Satyr Hoplite, Attack for 3
- Turn 3: Coordinated Assault, Attack for 7



# Cards In Hand

18



Option 2:

- Turn 1: Satyr Hoplite
- Turn 2: Foundry Street Denizen, Satyr Hoplite, Attack for 1?





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game

27



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 10



My life  
total: 10





# Sample Game

37



Opponent life  
total: 10



My life  
total: 10





# Sample Game



Opponent life  
total: 10



My life  
total: 10



# Sample Game



Opponent life  
total: 7



My life  
total: 10



# Sample Game



Opponent life  
total: 7



My life  
total: 10





# Sample Game



Opponent life  
total: 7



My life  
total: 10





# Sample Game



Opponent life  
total: 7



My life  
total: 10



# Sample Game



Opponent life  
total: 7



My life  
total: 10



# Sample Game



Opponent life  
total: 7



My life  
total: 10



# Block Analysis



## Single Block





# Block Analysis

46



## Single Block



# Block Analysis



## Double Block



# Block Analysis



## Double Block



# Sample Game



Opponent life total: 7



My life total: 10





# Sample Game

50



Opponent life  
total: 7



1



My life  
total: 8



2



# Sample Game



Opponent life total: 7



My life total: 8



# Sample Game



Opponent life total: 7



My life total: 8





# Sample Game



Opponent life total: 7



My life total: 8





# Sample Game



Opponent life  
total: 7



My life  
total: 8



# Sample Game



Opponent life total: 7



My life total: 8



# Sample Game



Opponent life total: 7



My life total: 8





# Sample Game



Opponent life  
total: 7



My life  
total: 8





# Sample Game



Opponent life total: 7



My life total: 8



# Sample Game



Opponent life total: 7



My life total: 8



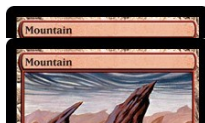
# Sample Game



Opponent life total: 7



My life total: 8





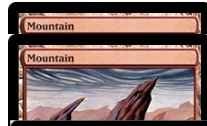
# Sample Game



Opponent life  
total: 7



My life  
total: 8





# Sample Game



Opponent life total: 7



My life total: 8



# Sample Game



Opponent life  
total: 7



My life  
total: 8



# Sample Game



Opponent life total: 7



My life total: 8





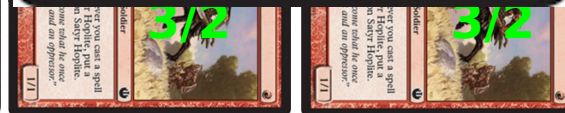
# Sample Game



Opponent life total: 7



My life total: 8





# Sample Game



Opponent life total: 7



My life total: 8



# Sample Game



Opponent life  
total: 7



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Sample Game



Opponent life  
total: 6



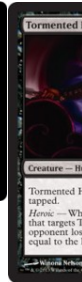
My life  
total: 8



# Sample Game



Opponent life  
total: 6



My life  
total: 8



creature an opponent controls can't block this turn.  
The Iron Games award no medals. Athletes vie for a visit from Irons, god of victory.  
2/1



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 8



# Block Analysis



Deals 2 damage.  
Receives 3  
damage



Deals 3 damage.  
Receives 2  
damage



# Block Analysis



# Sample Game



Opponent life  
total: 6



My life  
total: 8





# Sample Game



Opponent life  
total: 6



My life  
total: 6



# Sample Game



Opponent life  
total: 6



My life  
total: 6







# Sample Game



Opponent life  
total: 6



My life  
total: 6





# Sample Game



Opponent life  
total: 6



My life  
total: 6



# Sample Game



Opponent life  
total: 6



My life  
total: 6



# Sample Game



Opponent life  
total: 6



My life  
total: 6





# Attack Analysis

95



Opponent life  
total: 6

Option 1:

- Attack With Both our creatures, hope Bob doesn't block.
- Lightning bolt to kill Bob





# Attack Analysis

96



Opponent life  
total: 6

Option 1:

- Attack With Both our creatures, hope Bob doesn't block.
- Lightning bolt to kill Bob



Bob Blocks = Trade 2/1 for 1/1 and 3/2 for 2/2

# Attack Analysis

97



Opponent life  
total: 6

Option 2:

- Lightning Bolt  
Creature 1.
- Attack with  
War-Name  
Aspirant
- Leave Bob at 3



# Sample Game



Opponent life  
total: 6



My life  
total: 6





# Sample Game



Opponent life  
total: 6



My life  
total: 6



# Sample Game

10  
0



Opponent life  
total: 6



My life  
total: 6



# Sample Game



Opponent life  
total: 6



9



My life  
total: 6



0





# Sample Game

10  
2



Opponent life  
total: 6



My life  
total: 6

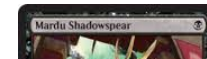
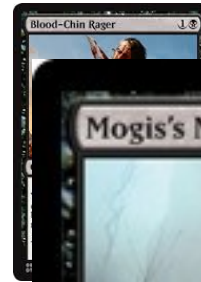


# Sample Game

10  
3



Opponent life  
total: 6



My life  
total: 6

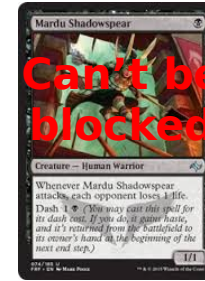


# Sample Game

10  
4



Opponent life  
total: 6



My life  
total: 6





# Sample Game



Opponent life  
total: 6



9



My life  
total: -2



0



# Why did we lose?

10  
6



- Playing against black, we should have been suspicious of bile blight



# Why did we lose?

10



- People playing these 3 cards are usually playing the fourth





# So What?

10

8



- Modelling our opponent's hand is important
- Guessing randomly doesn't help
- Can guess based on opponent plays

# Imperfect Info Games

10

9



- What are they?
- General Methods for creating
  - Abstraction
  - Sampling

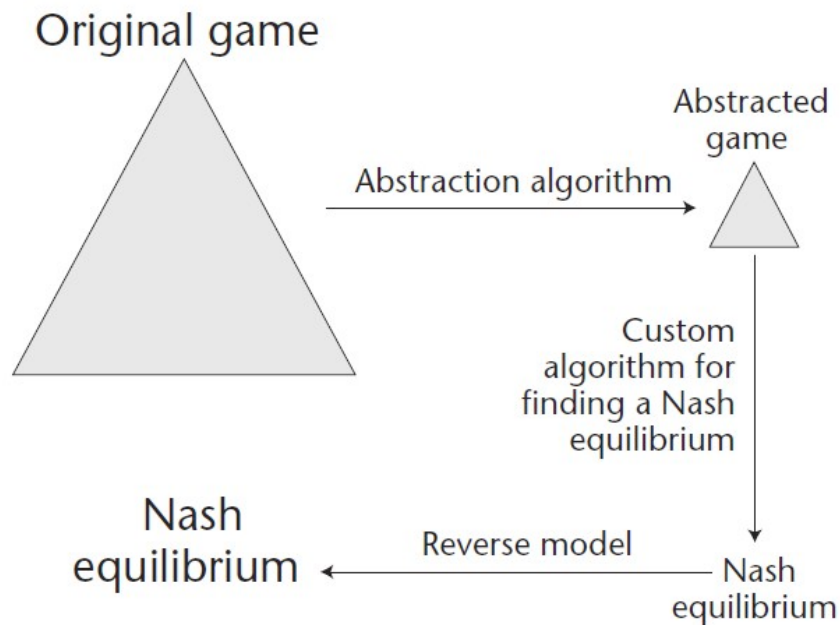
# Abstraction

11

0



- Information Abstraction
- Action Abstraction
- Phase Abstraction





# Information Abstraction

11

1



- Merge Similar States of the Game
- Agents can't distinguish between merged states
- Eg: Merge hands of similar strength into buckets
- Works with Poker, not so well with Bridge, Skat and MTG

# Action Abstraction

11

2



- Group Distinct Actions Together
- Works in Texas Hold'em
- Eg: Grouping betting actions

# Phase Abstraction

11  
3



- Some games are played in different phases
- Solve for the phases separately
- MTG seems like a good candidate



# Sampling Algorithms

11

4



- Traverse Selected Nodes In a Game Tree
- Monte Carlo Tree Search (MCTS)
  - Selection
  - Expansion
  - Simulation
  - Back Propagation

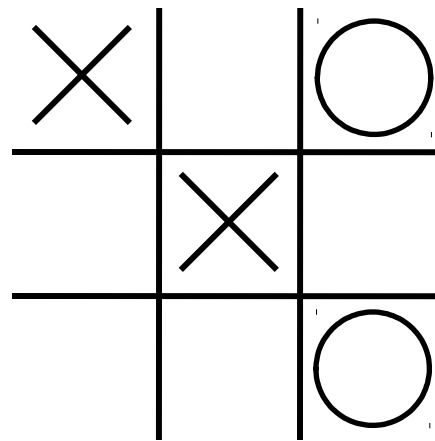
# MCTS Illustration



11

5

Play for X



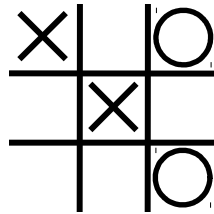
# MCTS Illustration

11

6



Selection



# MCTS Illustration

11

7



## Expansion

X	1	O
2	X	3
4	5	O

**5 Unexplored  
Children**



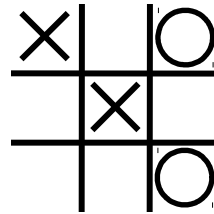
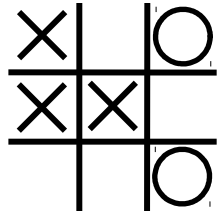
# MCTS Illustration

11

8



## Expansion



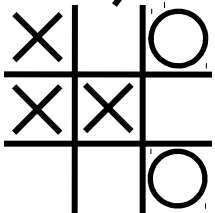
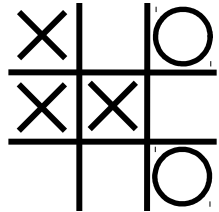
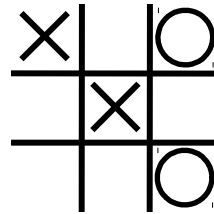
# MCTS Illustration

11

9



## Simulation



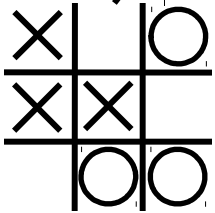
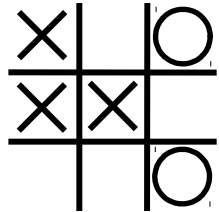
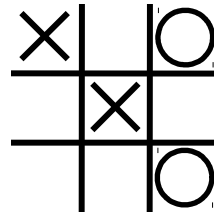
# MCTS Illustration

12

0



## Simulation



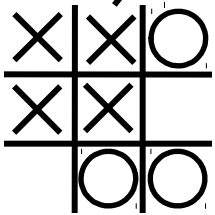
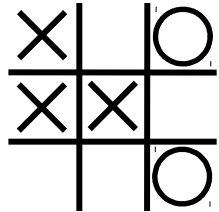
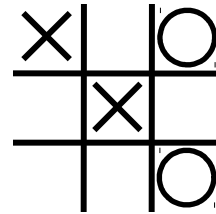
# MCTS Illustration

12

1



## Simulation

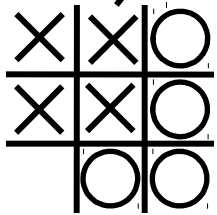
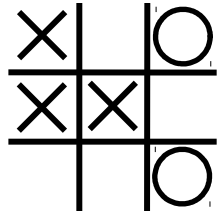
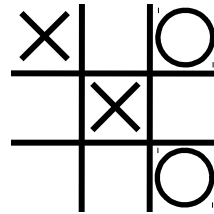




# MCTS Illustration



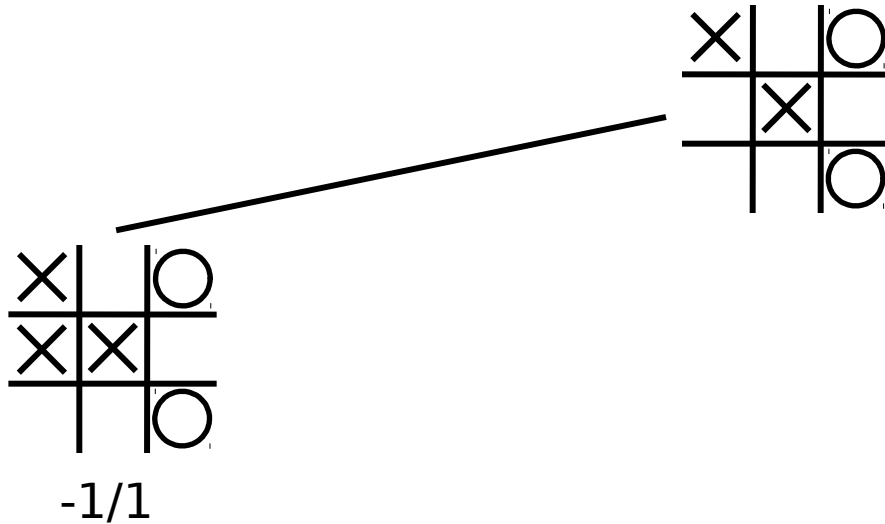
## Simulation



# MCTS Illustration



## Backpropagation



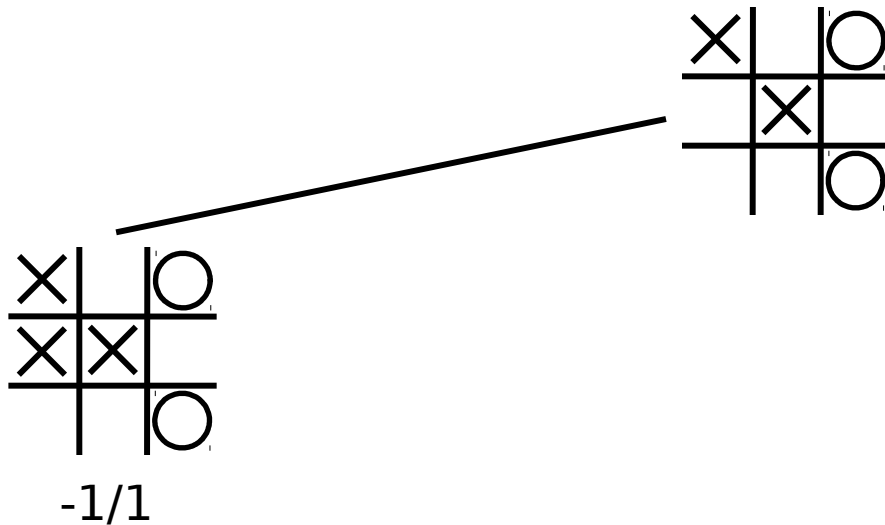
# MCTS Illustration

12

4



## Backpropagation

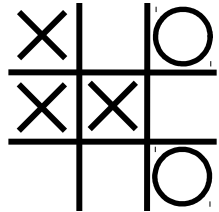


Iteration 1 Complete

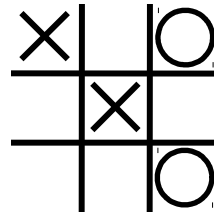
# MCTS Illustration



## Selection



-1/1





# MCTS Illustration



## Expansion

X		O
X	X	
		O

-1/1

X	1	O
	X	2
3	4	O

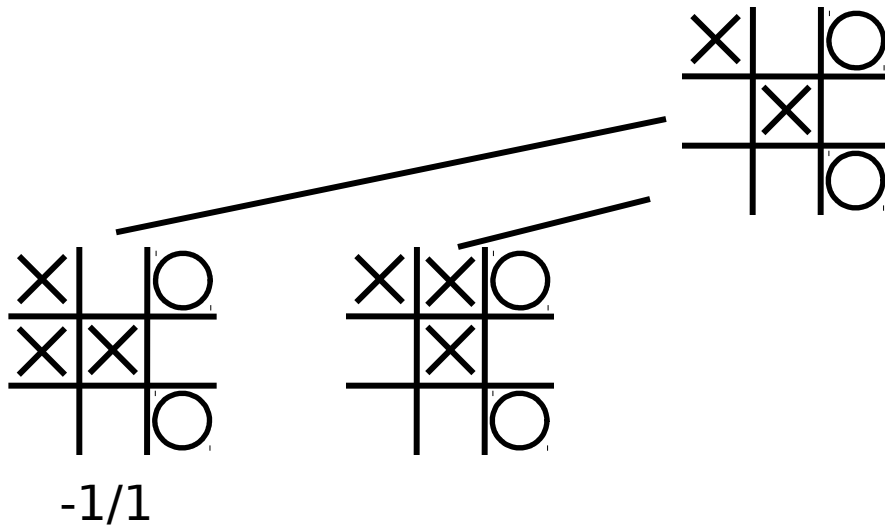
4

**Unexplored Children**

# MCTS Illustration



## Expansion



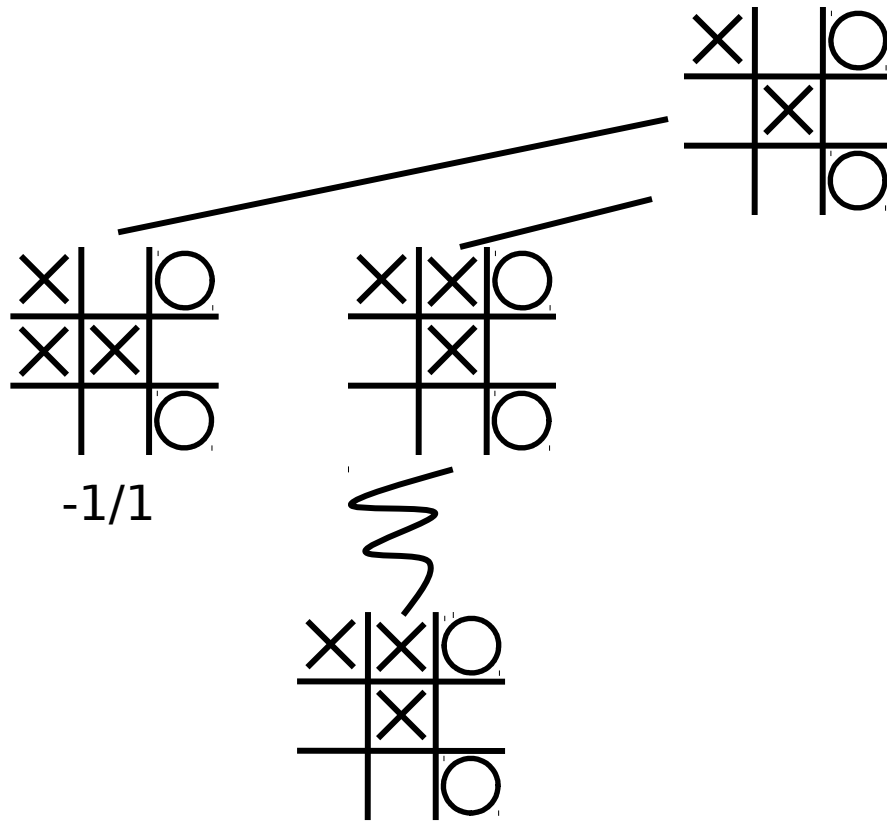
# MCTS Illustration

12

8



## Simulation



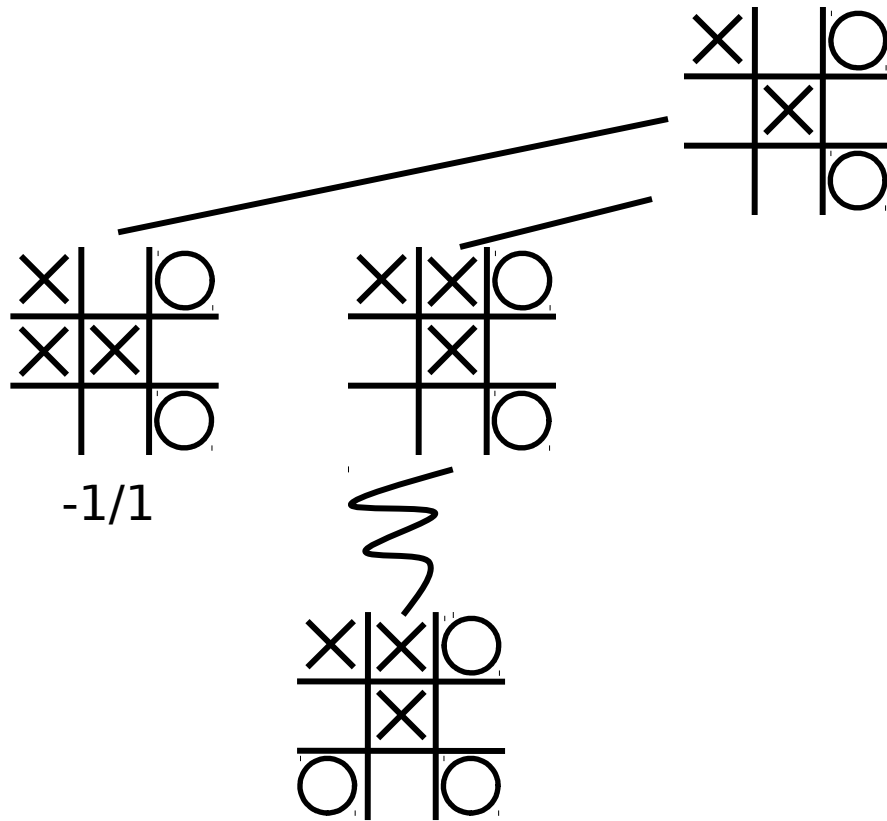
# MCTS Illustration

12

9



## Simulation

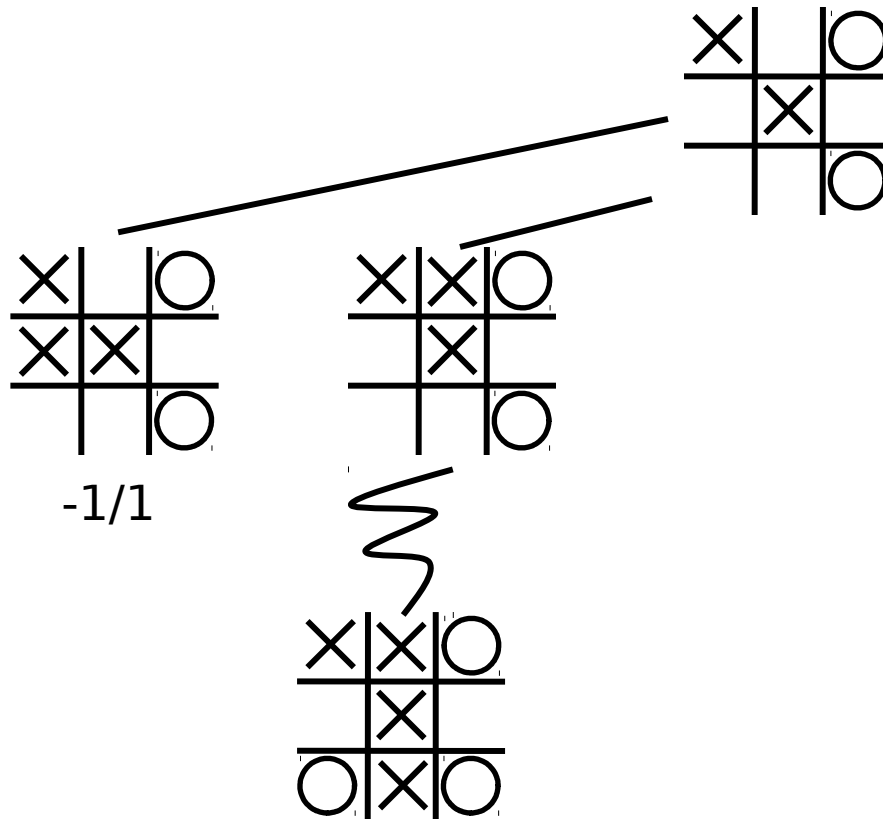




# MCTS Illustration



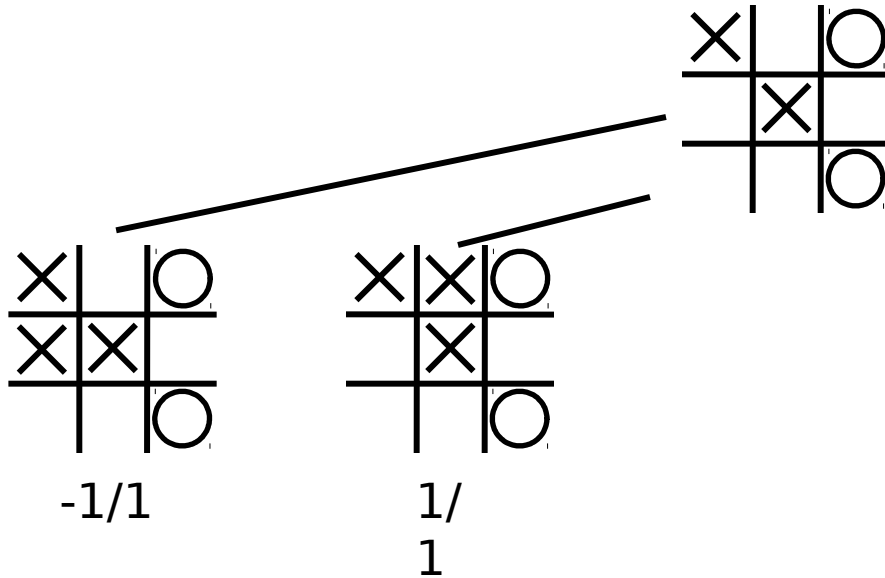
## Simulation



# MCTS Illustration



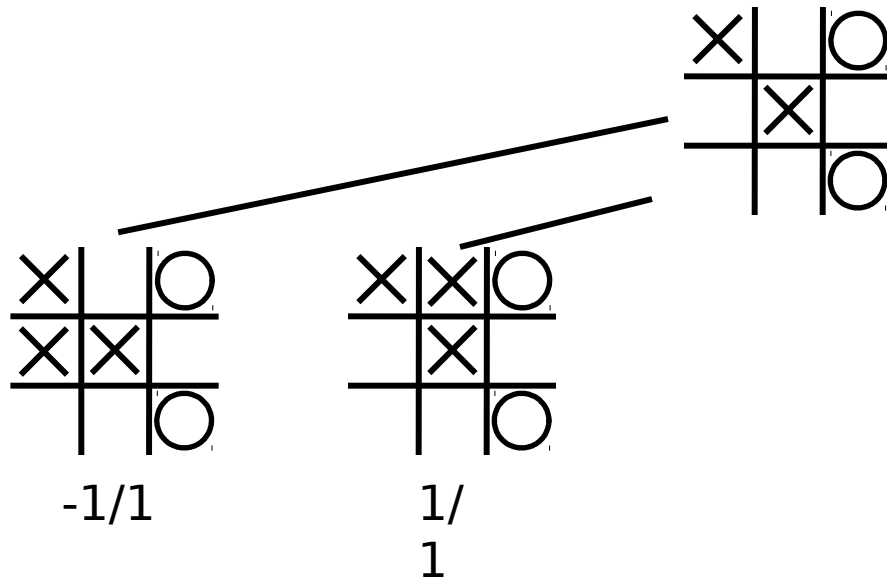
## Backpropagation



# MCTS Illustration

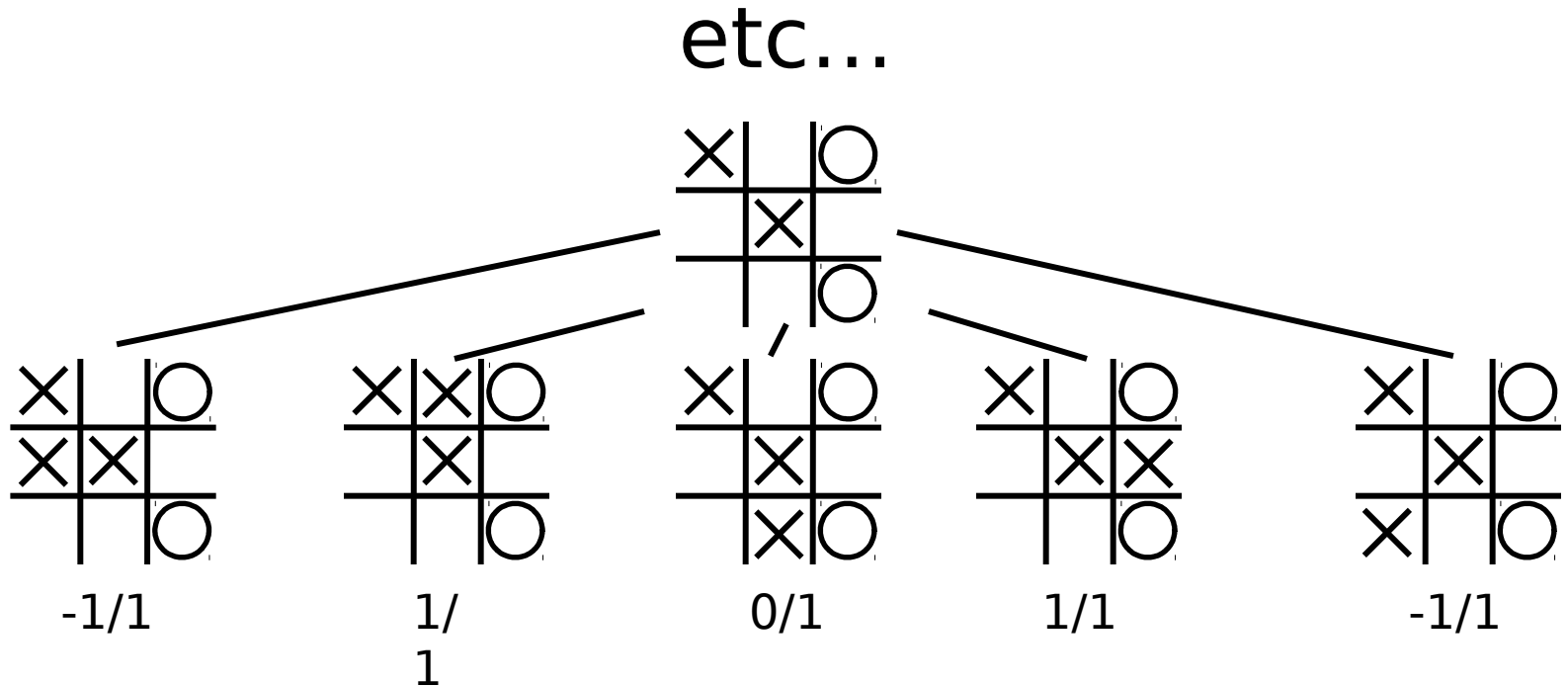


## Backpropagation



Iteration 2 Complete

# MCTS Illustration



Iteration 5 complete



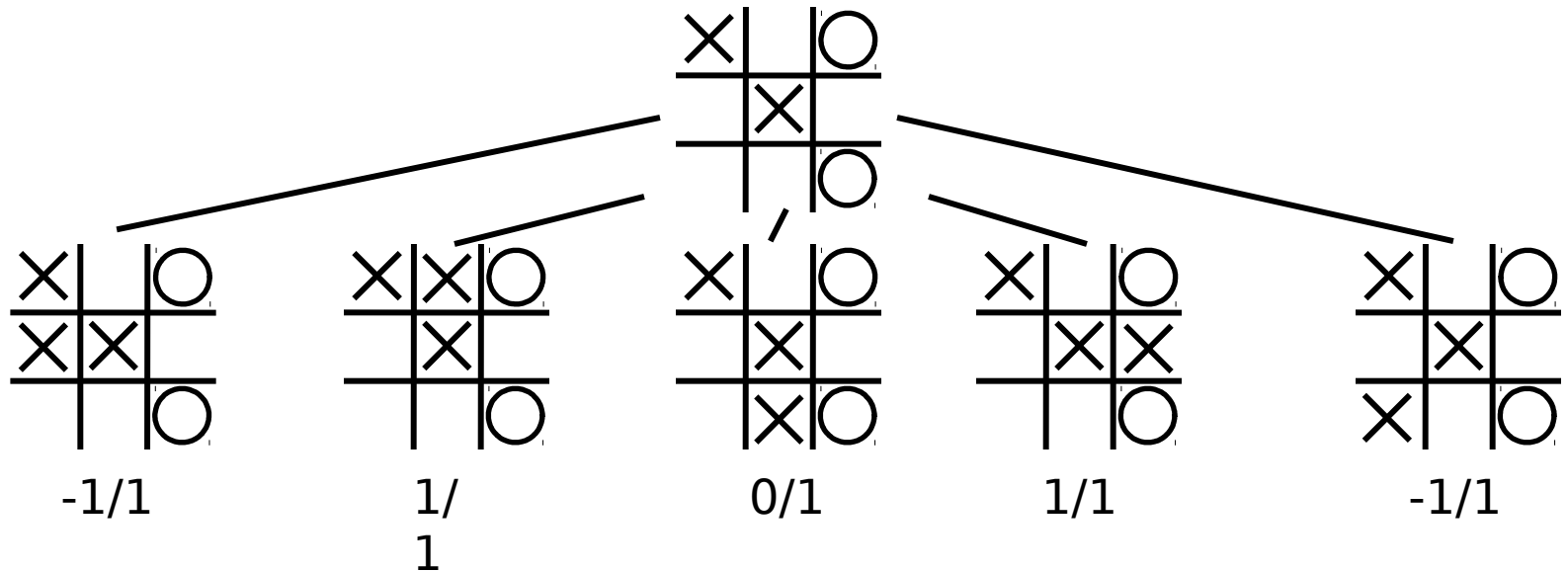
# MCTS Illustration

13

4



## Selection

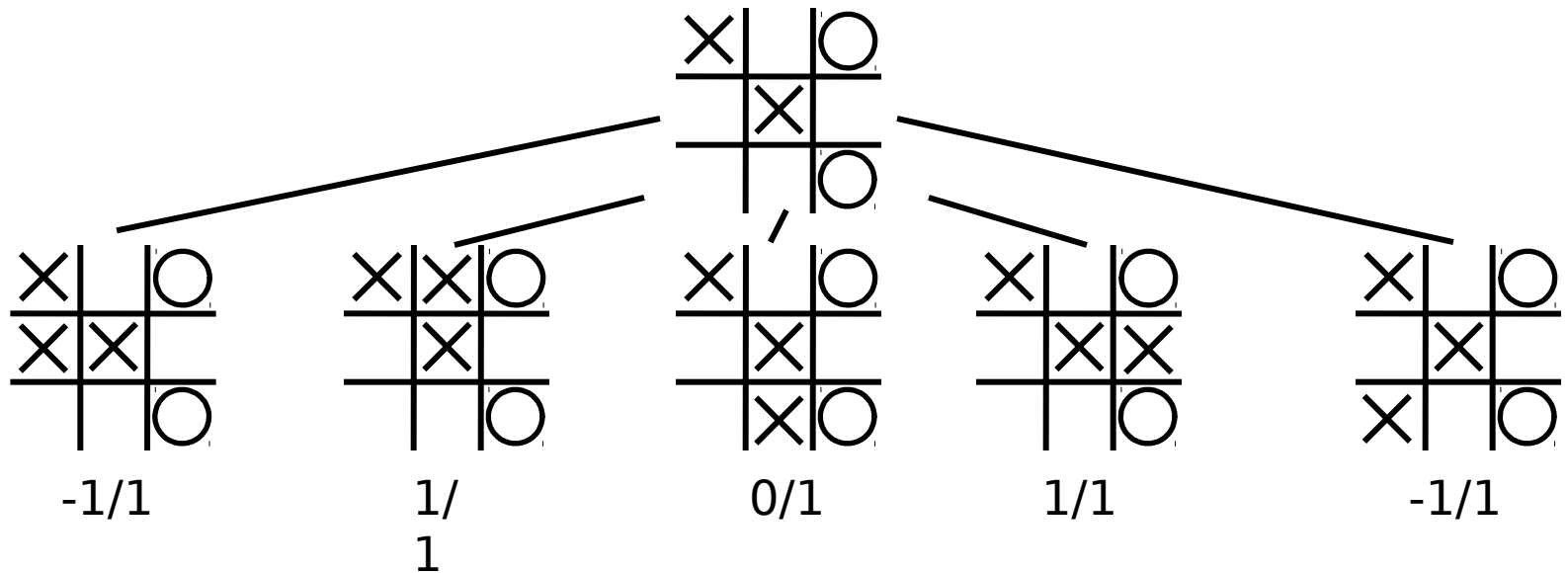


$$\text{UCB: } \bar{x}_j + \sqrt{\frac{2 \ln(n)}{n_j}}$$

# MCTS Illustration



## Selection



$$-1 + \sqrt{\frac{2 \ln(5)}{1}} = 0.7941$$

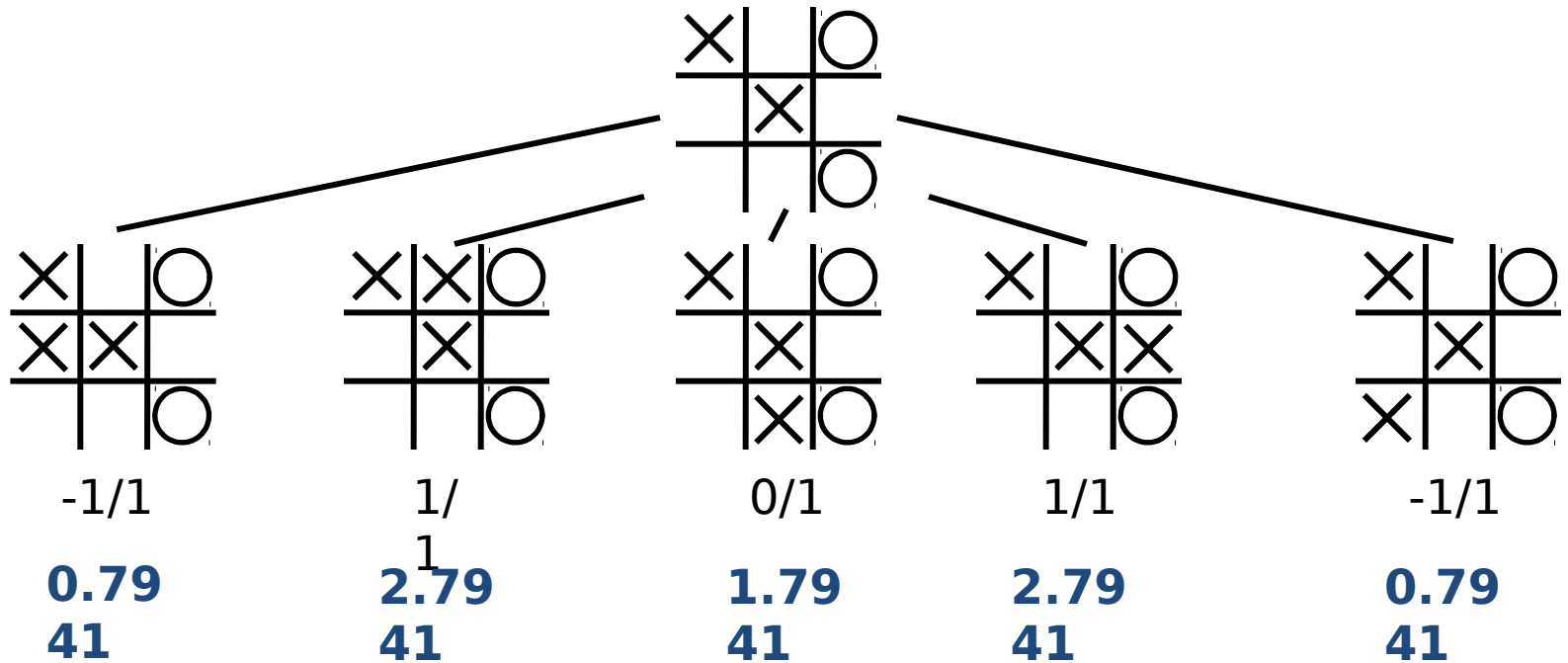
# MCTS Illustration

13

6



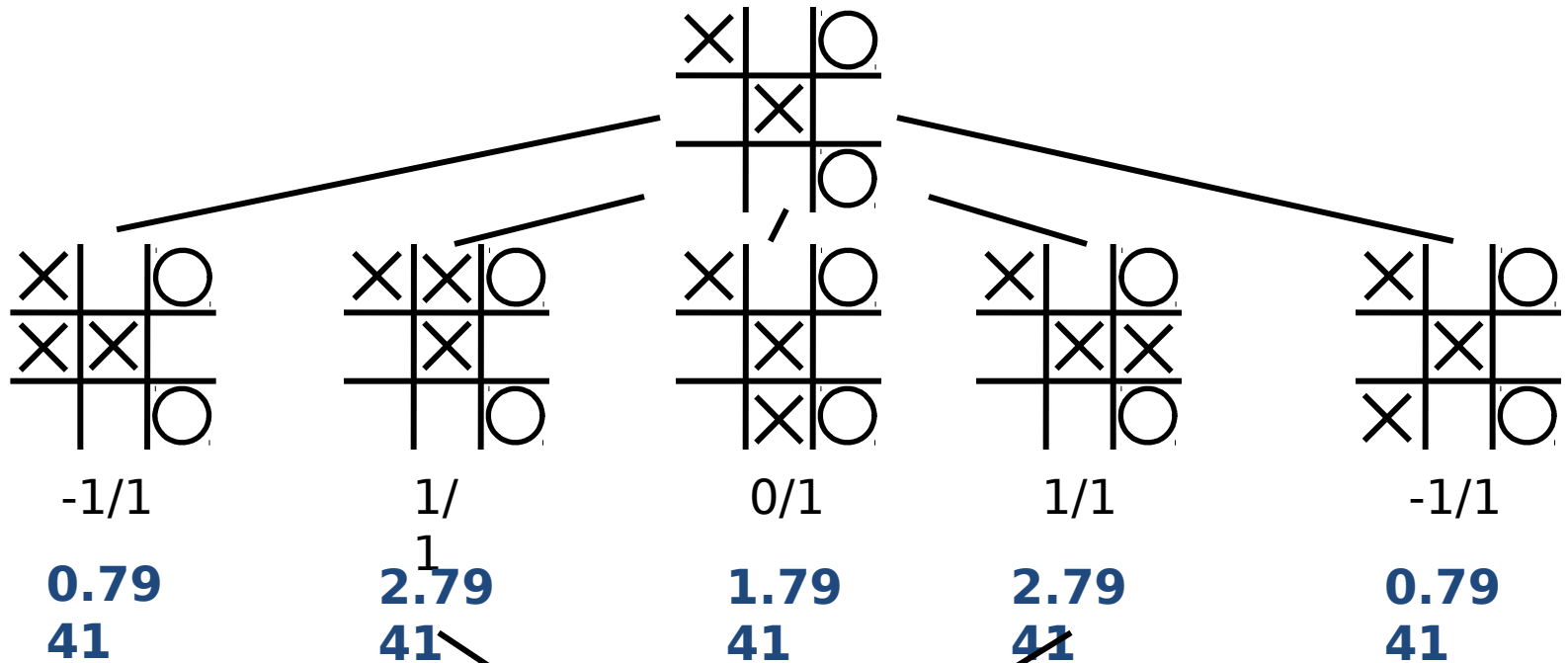
## Selection



# MCTS Illustration



## Selection



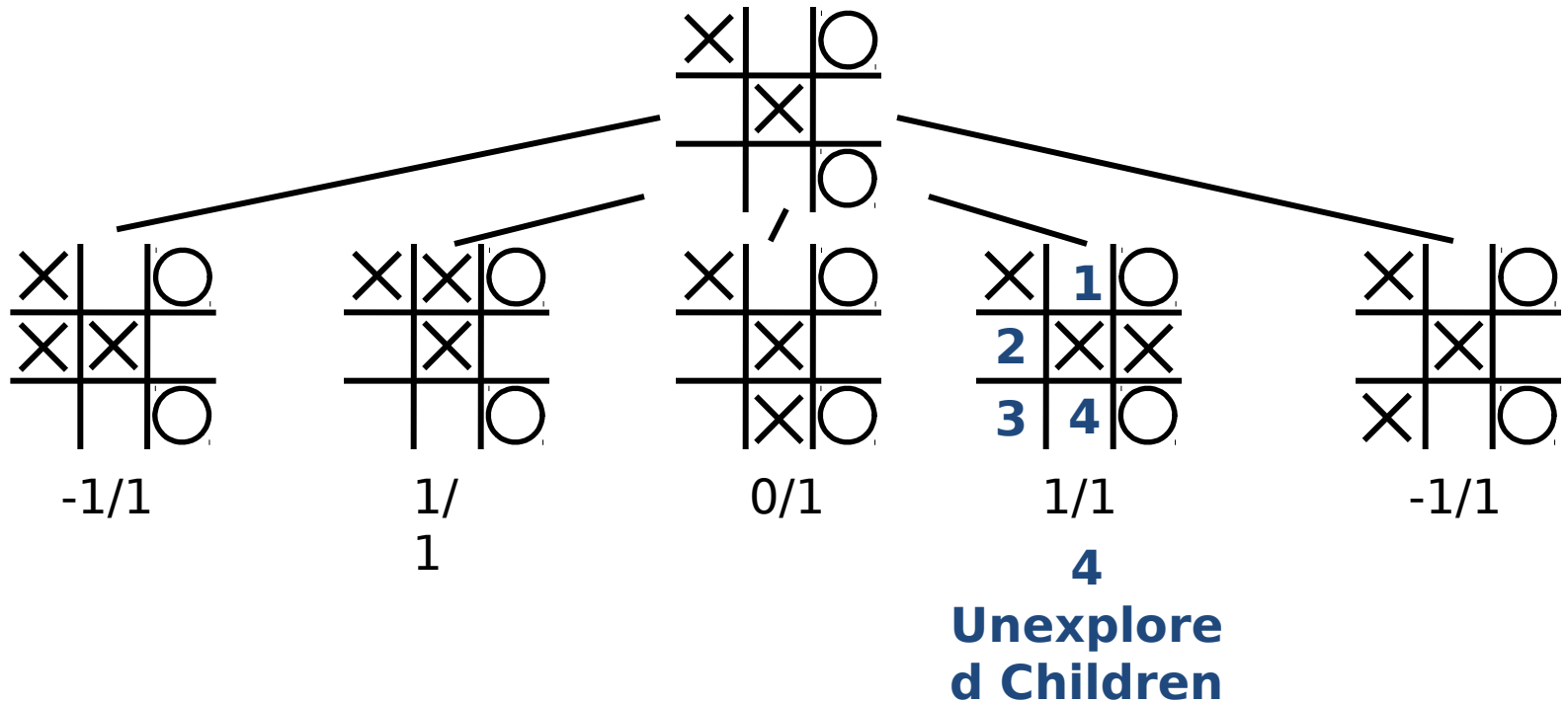
Tie! Break Tie  
Randomly



# MCTS Illustration



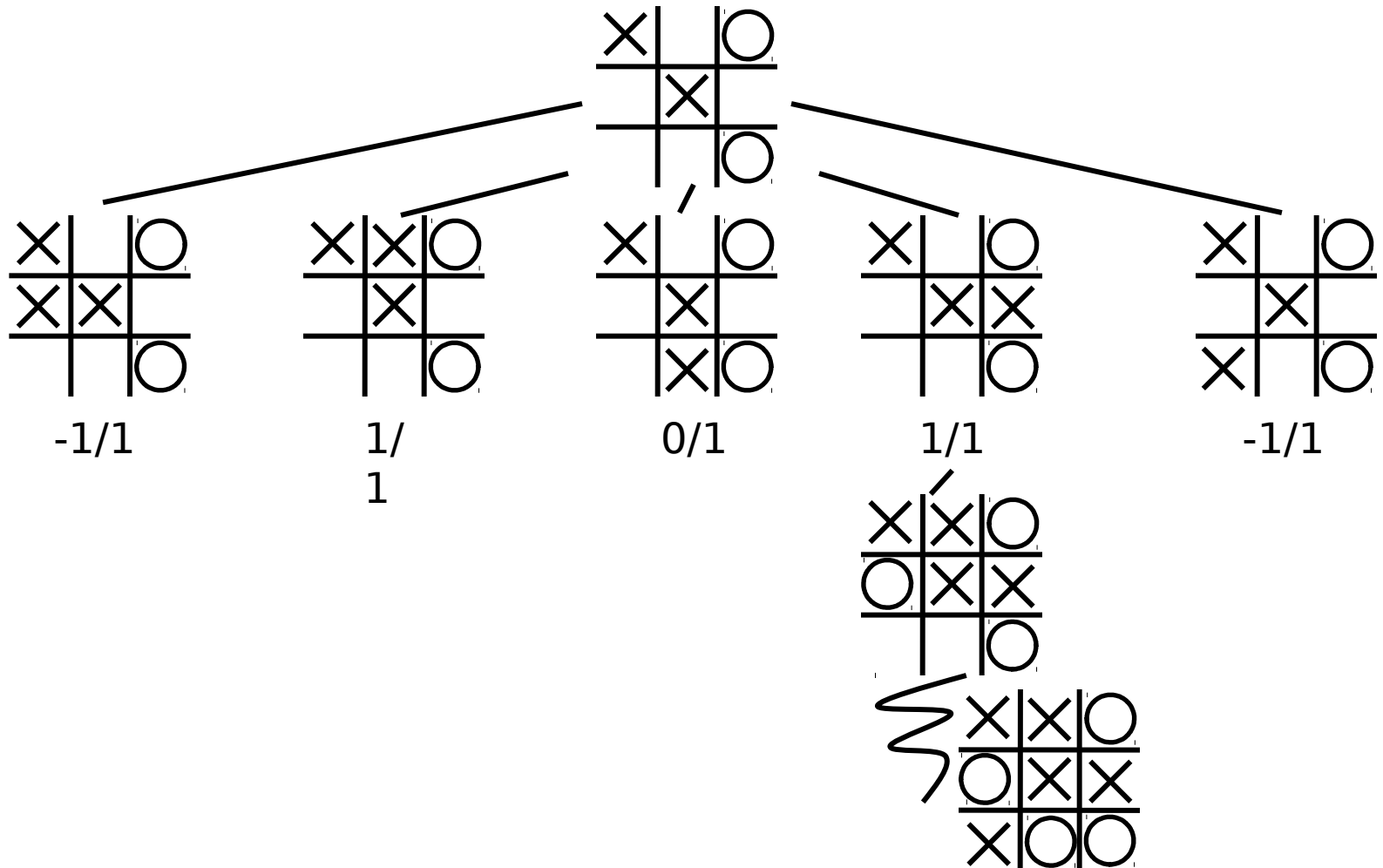
## Expansion



# MCTS Illustration



## Simulation



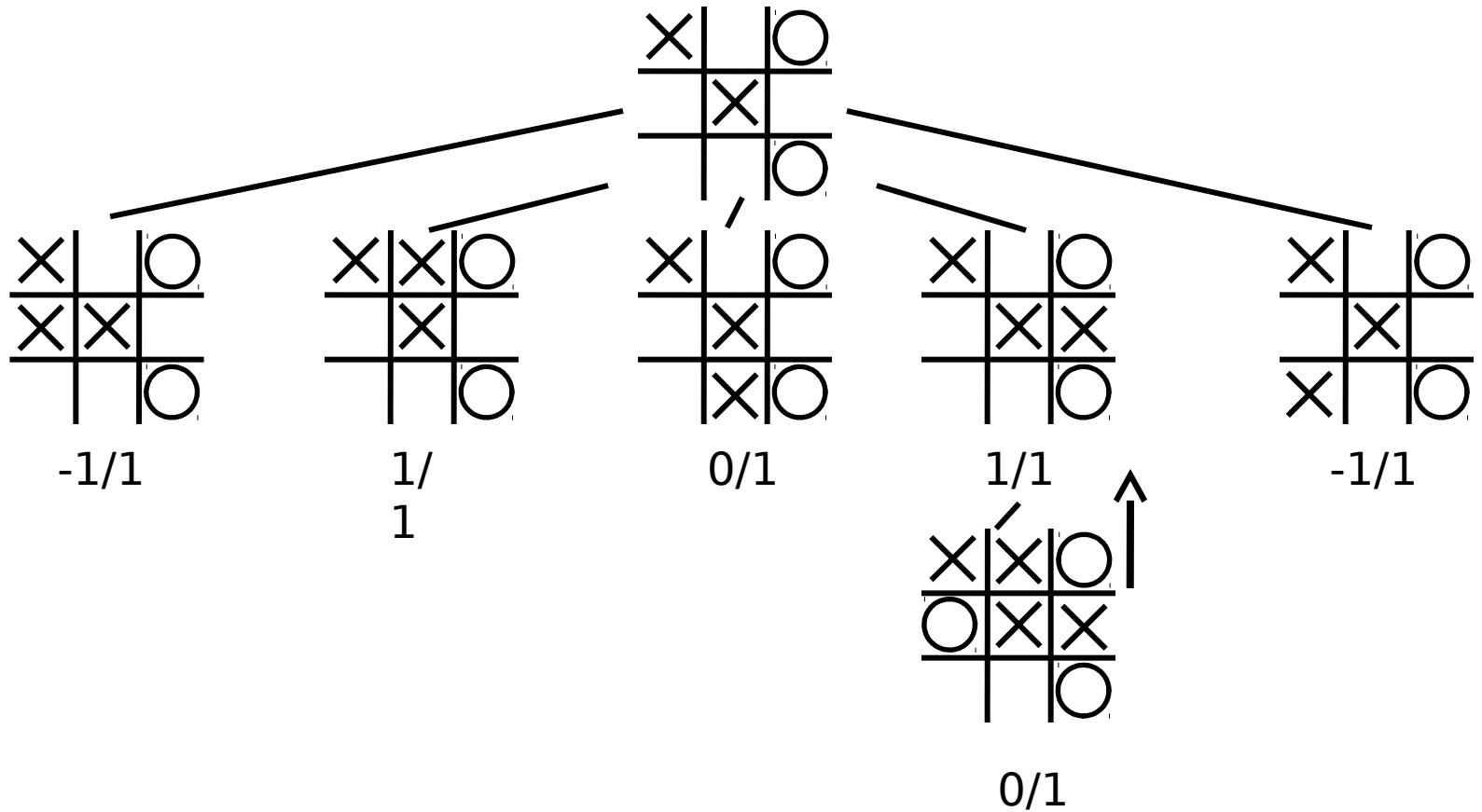
# MCTS Illustration

14

0



## Backpropagation



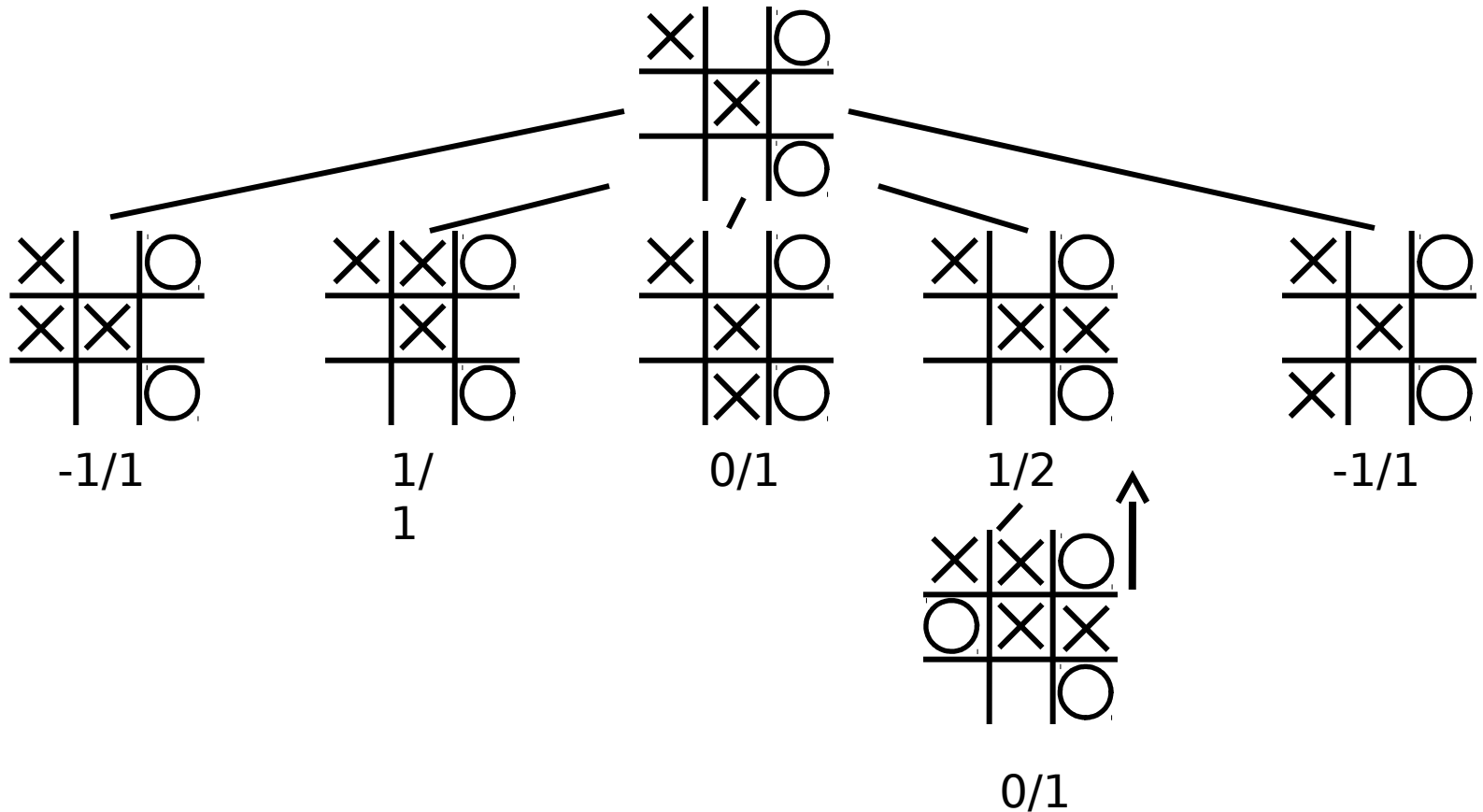
# MCTS Illustration

14

1



## Backpropagation





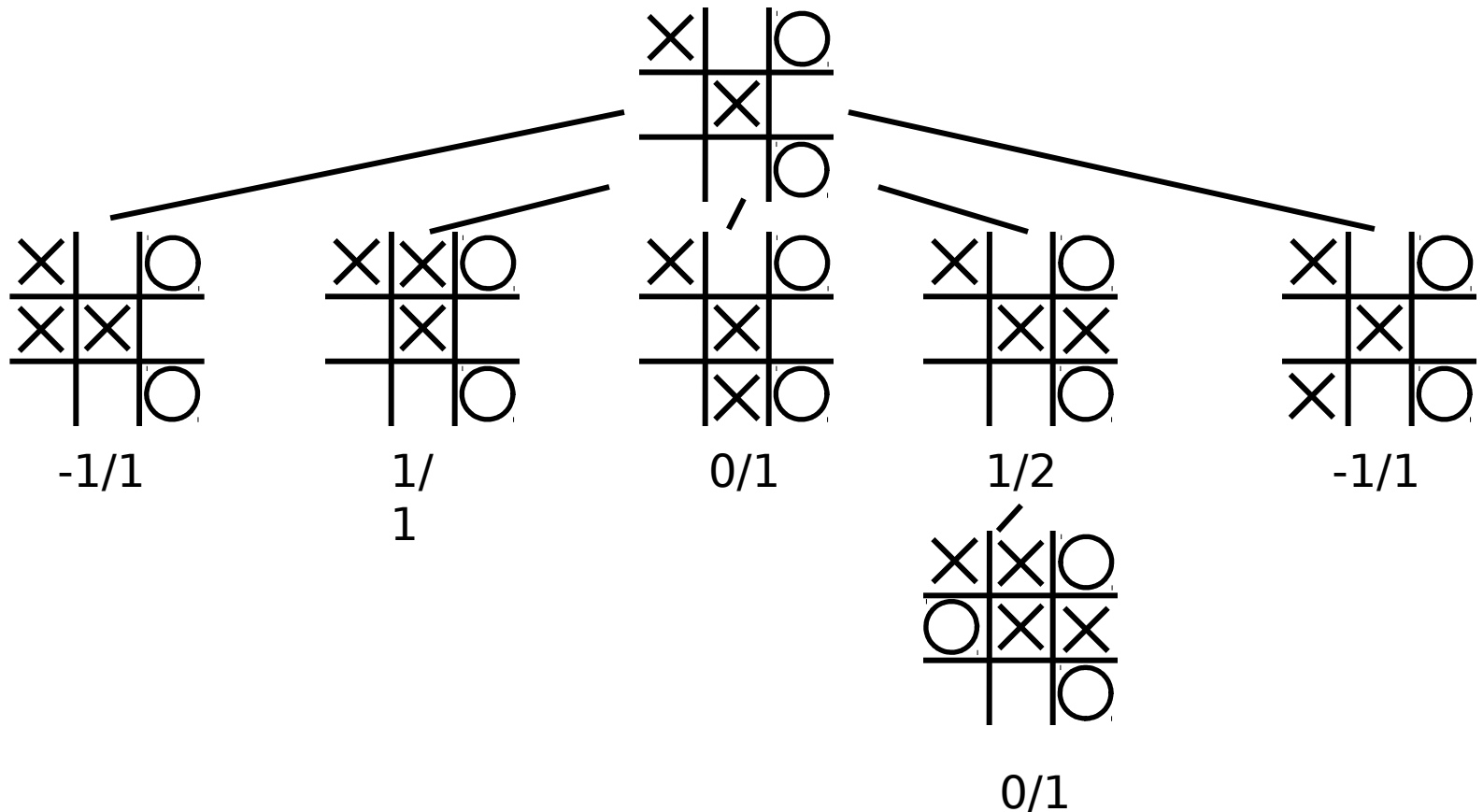
# MCTS Illustration

14

2



## Backpropagation



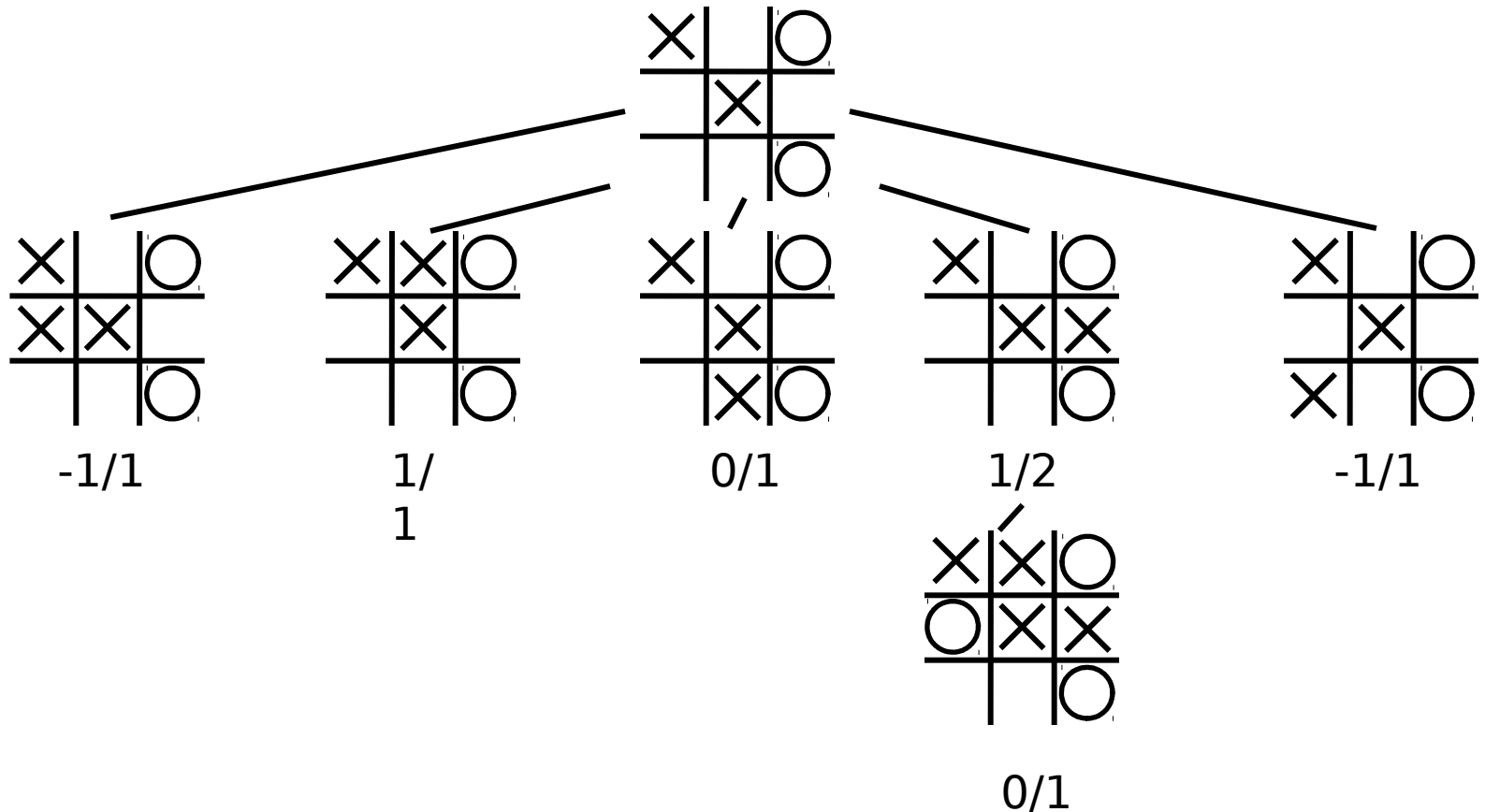
Iteration 6 complete

# MCTS Illustration

14  
3



Continue until some predefined limit...



# MCTS For MTG

14

4



- Peter Cowling
- Perfect Information Monte Carlo Search (PIMC)
  - Sample Possible Worlds
  - Find Best Move From Worlds
  - Play most chosen move

# PIMC For MTG

14

5



- Guess opponent's hand
- Guess exactly how our opponent will draw.
- Guess exactly how we will draw
- Collection of guesses makes a "World"
- Run MCTS on the "World"
- Repeat above



# Cowling et al MTG

14

6



- Used PIMC
- MCTS used Strong Heuristic Player + Random
- Downsides:
  - Need to update rules for the agent when card base changes
  - Provided opponent with exact composition of opponent's deck (Unrealistic)

# Our Approach

14



- Use a sampling algorithm
- Use Disambiguation to gain knowledge
- Use the knowledge to predict opponent's hand
- Random Predictions will lead to bad estimates/bad play
- Better estimate = Reduced # states => Better sampling

# Card Correlation

14

8



- What does this mean?
- Required Reasonable Decks
  - Required Reasonable Agents
  - Deck Building Procedure
- Play off against pre made deck
- Take top 1000 decks, create card correlation matrix

# Card Correlation



	LG	C1G	C2G	IG	LU	C1U	C2U	IU	LR	C1R	C2R	IR	LW	C1W	C2W	IW	LB	C1B	C2B	IB
LG	10.7839	2.4678	5.0594	4.6664	-2.7898	-0.6287	-1.4409	-1.2534	-2.5365	-0.6145	-1.1482	-1.1208	-2.7117	-0.5753	-1.269	-1.1018	-2.746	-0.6466	-1.3247	-1.0698
C1G	2.4678	8.2908	1.2272	0.0372	-0.6627	-0.5786	-0.9977	-0.8906	-0.5494	-0.6699	-0.813	-0.7926	-0.6775	-0.6333	-0.9041	-1.0028	-0.5783	-0.6518	-0.8288	-0.7921
C2G	5.0594	1.2272	12.1429	2.1016	-1.1936	-0.8562	-1.7037	-1.3069	-1.2032	-0.8707	-1.6765	-1.3733	-1.3441	-0.9978	-1.6685	-1.2199	-1.3185	-0.8101	-1.6924	-1.2958
IG	4.6664	0.0372	2.1016	12.6405	-1.1538	-0.864	-1.4779	-1.5416	-1.1087	-0.8669	-1.1813	-1.6215	-1.0933	-0.8831	-1.2192	-1.2959	-1.3106	-0.9426	-1.3507	-1.5348
LU	-2.7898	-0.6627	-1.1936	-1.1538	10.6839	2.4392	4.8019	4.7244	-2.7319	-0.5665	-1.1659	-1.2786	-2.5315	-0.6214	-1.1042	-1.2204	-2.6307	-0.5758	-1.1186	-1.304
C1U	-0.6287	-0.5786	-0.8562	-0.864	2.4392	8.4824	1.1396	-0.0056	-0.6722	-0.6803	-0.9666	-0.8665	-0.4886	-0.611	-0.8216	-0.9035	-0.6497	-0.6342	-0.8631	-0.9709
C2U	-1.4409	-0.9977	-1.7037	-1.4779	4.8019	1.1396	12.1412	2.1208	-1.1201	-0.8034	-1.479	-1.0741	-1.0917	-0.9473	-1.7921	-1.3093	-1.1492	-0.8435	-1.7366	-1.2372
IU	-1.2534	-0.8906	-1.3069	-1.5416	4.7244	-0.0056	2.1208	12.6595	-1.3088	-1.0117	-1.2254	-1.6586	-0.995	-0.8267	-1.0572	-1.4848	-1.1673	-0.8635	-1.2479	-1.6597
LR	-2.5365	-0.5494	-1.2032	-1.1087	-2.7319	-0.6722	-1.1201	-1.3088	10.4879	2.3942	4.8269	4.7135	-2.6492	-0.6047	-1.2422	-1.1756	-2.5703	-0.5596	-1.2608	-1.1294
C1R	-0.6145	-0.6699	-0.8707	-0.8669	-0.5665	-0.6803	-0.8034	-1.0117	2.3942	8.3779	1.1666	0.2061	-0.6345	-0.6911	-0.9157	-0.9066	-0.5786	-0.6235	-0.8574	-0.8534
C2R	-1.1482	-0.813	-1.6765	-1.1813	-1.1659	-0.9666	-1.479	-1.2254	4.8269	1.1666	12.248	2.1329	-1.1887	-0.9099	-1.749	-1.3778	-1.3242	-0.9665	-1.9248	-1.2779
IR	-1.1208	-0.7926	-1.3733	-1.6215	-1.2786	-0.8665	-1.0741	-1.6586	4.7135	0.2061	2.1329	12.5473	-1.2298	-0.9487	-1.2944	-1.6145	-1.0844	-0.8153	-1.2609	-1.5658
LW	-2.7117	-0.6775	-1.3441	-1.0933	-2.5315	-0.4886	-1.0917	-0.995	-2.6492	-0.6345	-1.1887	-1.2298	10.8988	2.542	4.9253	4.7042	-3.0064	-0.785	-1.2976	-1.3457
C1W	-0.5753	-0.6333	-0.9978	-0.8831	-0.6214	-0.611	-0.9473	-0.8267	-0.6047	-0.6911	-0.9099	-0.9487	2.542	8.7958	1.0607	0.1636	-0.7406	-0.8177	-0.8409	-0.9126
C2W	-1.269	-0.9041	-1.6685	-1.2192	-1.1042	-0.8216	-1.7921	-1.0572	-1.2422	-0.9157	-1.749	-1.2944	4.9253	1.0607	12.0402	2.182	-1.3099	-0.9378	-1.5696	-1.3537
IW	-1.1018	-1.0028	-1.2199	-1.2959	-1.2204	-0.9035	-1.3093	-1.4848	-1.1756	-0.9066	-1.3778	-1.6145	4.7042	0.1636	2.182	12.3901	-1.2063	-0.8879	-1.2378	-1.495
LB	-2.746	-0.5783	-1.3185	-1.3106	-2.6307	-0.6497	-1.1492	-1.1673	-2.5703	-0.5786	-1.3242	-1.0844	-3.0064	-0.7406	-1.3099	-1.2063	10.9535	2.567	5.0017	4.8489
C1B	-0.6466	-0.6518	-0.8101	-0.9426	-0.5758	-0.6342	-0.8435	-0.8635	-0.5596	-0.6235	-0.9665	-0.8153	-0.785	-0.8177	-0.9378	-0.8879	2.567	8.4919	1.112	0.1903
C2B	-1.3247	-0.8288	-1.6924	-1.3507	-1.1186	-0.8631	-1.7366	-1.2479	-1.2608	-0.8574	-1.9248	-1.2609	-1.2976	-0.8409	-1.5696	-1.2378	5.0017	1.112	12.1566	2.1423
IB	-1.0698	-0.7921	-1.2958	-1.5348	-1.304	-0.9709	-1.2372	-1.6597	-1.1294	-0.8534	-1.2779	-1.5658	-1.3457	-0.9126	-1.3537	-1.495	4.8489	0.1903	2.1423	12.6162

L	Land
C1	Creature 1
C2	Creature 2
I	Instant

G	Green
U	Blue
R	Red
W	White
B	Black

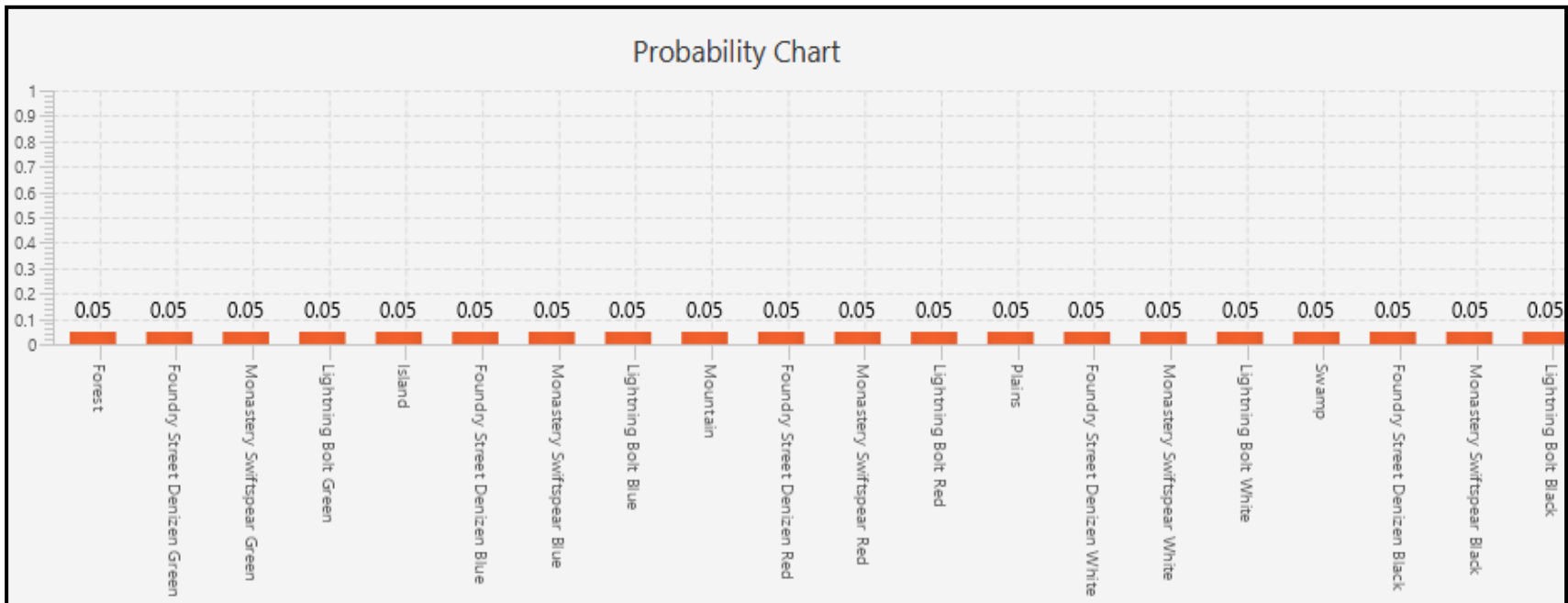
# Prior Knowledge Agent

15

0



- Should do
  - Better than no knowledge
  - Worse than perfect knowledge





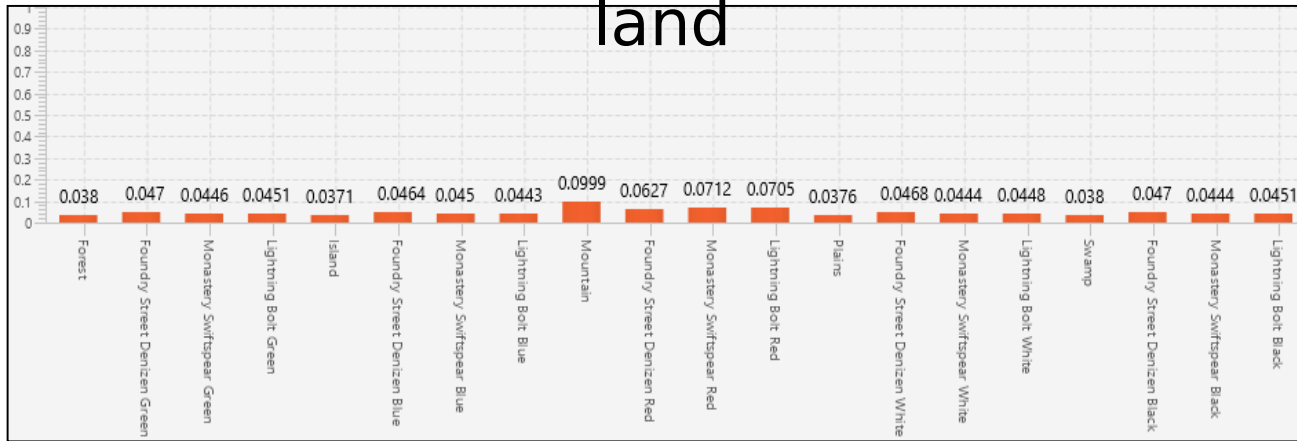
# Prior Knowledge Agent

15

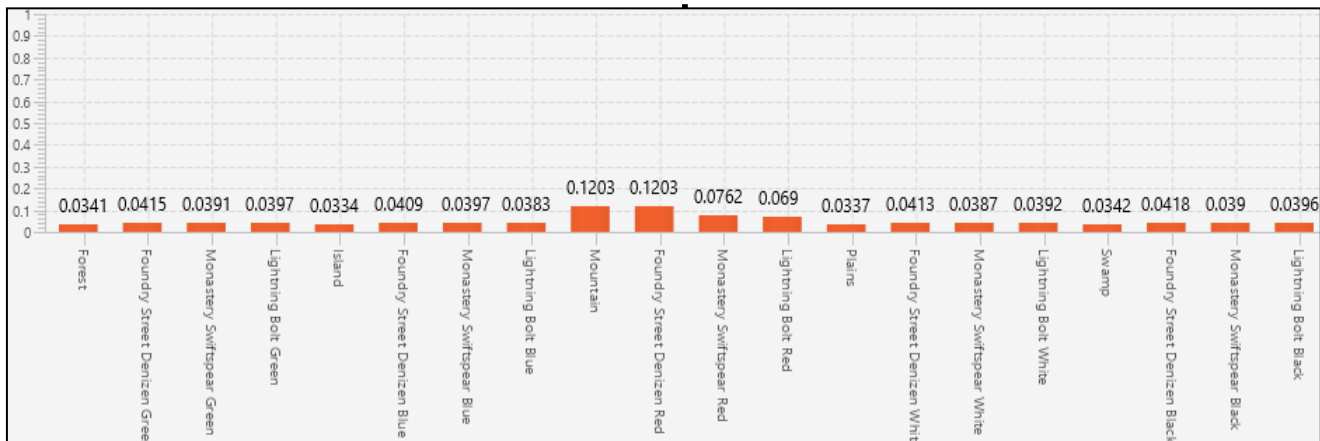
1



## Distribution After seeing a red land



## Distribution After seeing a red land, red

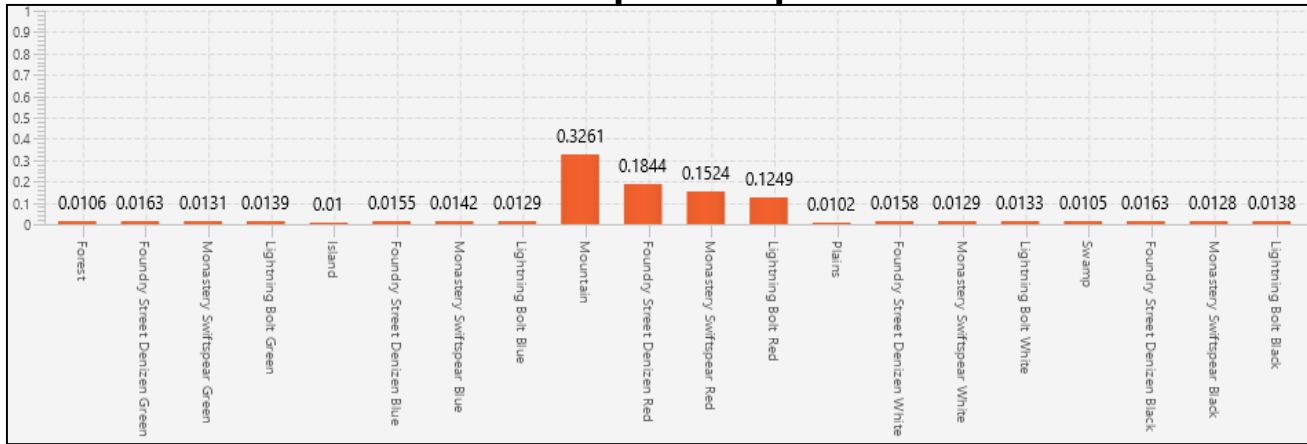


# Prior Knowledge Agent

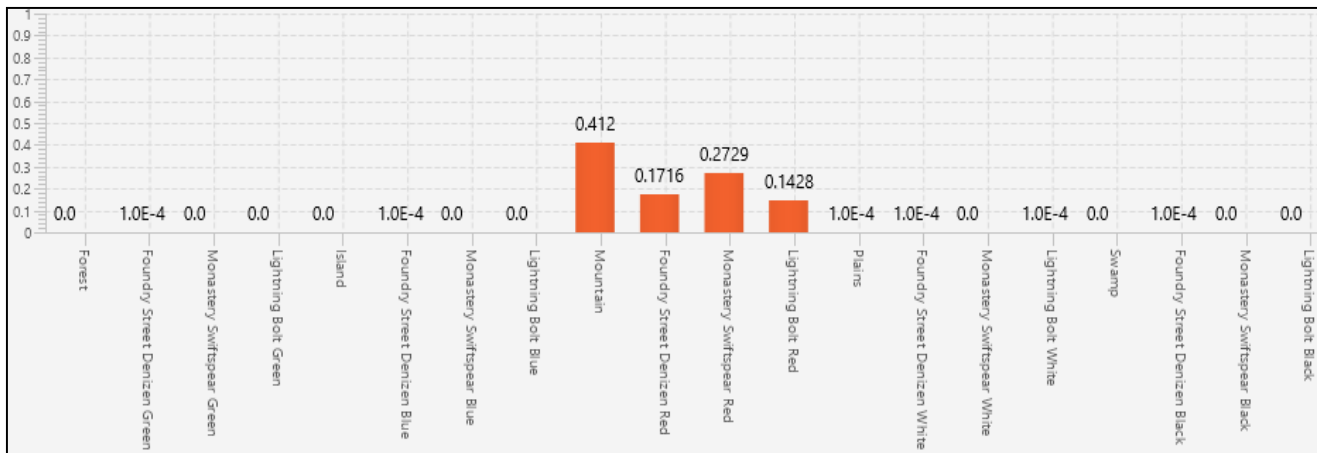
15  
2



Distribution After seeing a few more

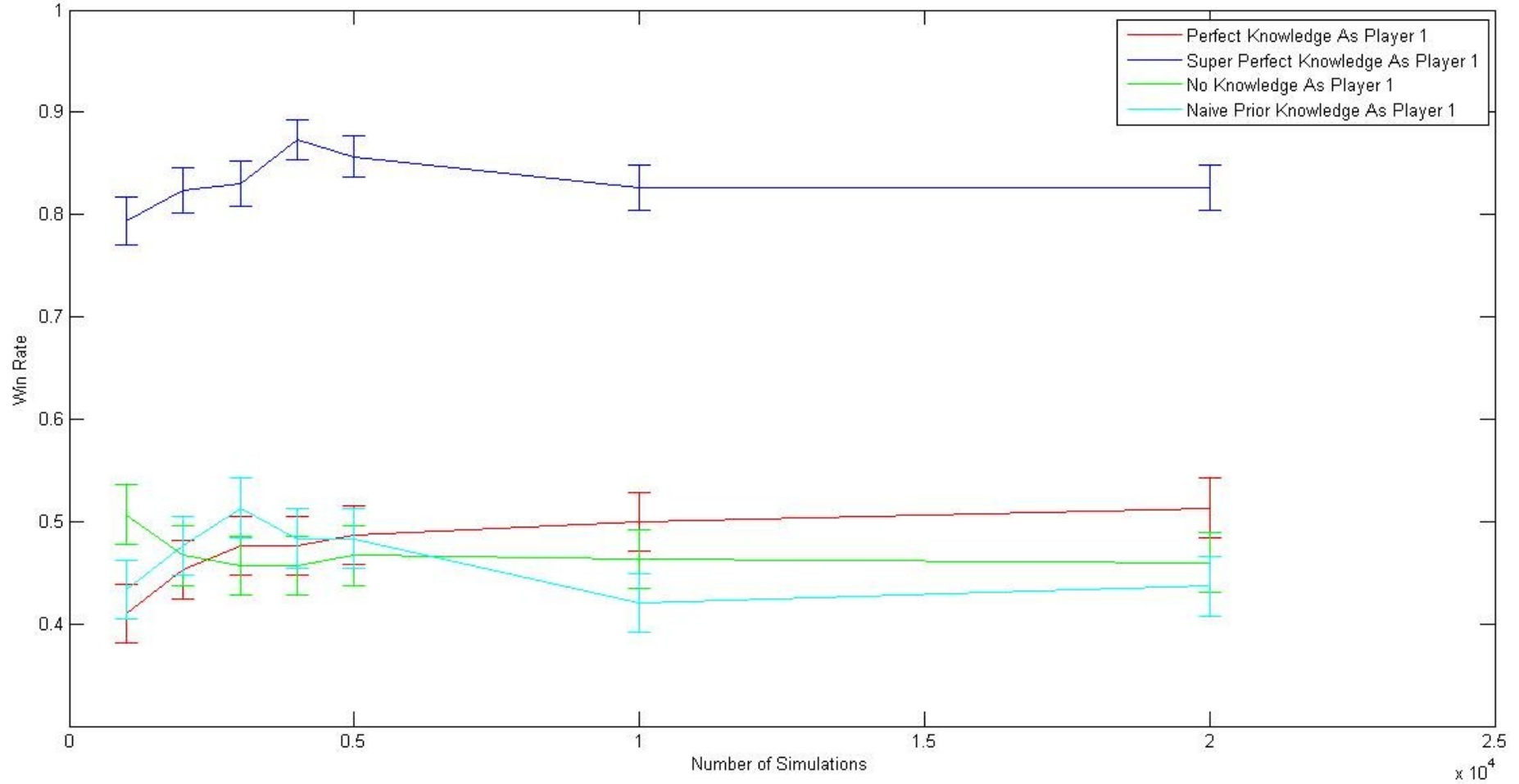


Eventually...



# Current Results

15  
3



# Future Work

15

4



- Carry Knowledge Across Games
- Run Experiments With Different Decks
- Model Distributions Over Decks Instead of cards